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2 Modeling of Multi-Functional Single Converter FACTS in Power Flow Analysis

This chapter discusses the recent developments in modeling of multi-functional single converter FACTS-devices in power flow analysis. The objectives of this chapter are:

1. to model the well-recognized FACTS devices such as STATOM, SVC, SSSC and TCSC in power flow analysis,
2. to establish multi-control functional models of these FACTS-devices,
3. to handle various internal and external operating constraints of FACTS-devices.

2.1 Power Flow Calculations

2.1.1 Power Flow Methods

It is well known that power flow calculations are the most frequently performed routine power network calculations, which can be used in power system planning, operational planning, and operation/control. It is also considered as the fundamental of power system network calculations. The calculations are required for the analysis of steady-state as well as dynamic performance of power systems.

In the past, various power flow solution methods such as impedance matrix methods, Newton-Raphson methods, decoupled Newton power flow methods, etc have been proposed [1]-[13]. Among the power flow methods proposed, the Newton's methods using sparse matrix elimination techniques [14] have been considered as the most efficient power flow solution techniques for large-scale power system analysis. A detailed review of power flow methods can be found in [1]. In this chapter, FACTS models for power flow analysis as well as the implementation of these models in Newton power flow will be discussed in detail.

2.1.2 Classification of Buses

In power flow analysis, all buses can be classified into the following categories:

Slack bus. At a slack bus, the voltage angle and magnitude are specified while the active and reactive power injections are unknown. The voltage angle of the slack

bus is taken as the reference for the angles of all other buses. Usually there is only one slack bus in a system. However, in some production grade programs, it may be possible to include more than one bus as distributed slack buses.

PV Buses. At a PV bus, the active power injection and voltage magnitude are specified while the voltage angle and reactive power injection are unknown. Usually buses of generators, synchronous condensers are considered as PV buses.

PQ Buses. At a PQ bus, the active and reactive power injections are specified while the voltage magnitude and angle at the bus are unknown. Usually a load bus is considered as a PQ bus.

2.1.3 Newton-Raphson Power Flow in Polar Coordinates

The Newton-Raphson Power Flow can be formulated either in polar coordinates or in rectangular coordinates. In this chapter, the implementation of FACTS models in the Newton-Raphson power flow will be discussed since the popularity of the methods. Basically, the Newton-Raphson power flow equations in polar coordinates may be given by [1]:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} -\Delta P \\ -\Delta Q \end{bmatrix} \quad (2.1)$$

where ΔP and ΔQ are bus active and reactive power mismatches while θ and V are bus magnitude and angle, respectively.

2.2 Modeling of Multi-Functional STATCOM

In recent years, energy, environment, deregulation of power utilities have delayed the construction of both generation facilities and new transmission lines. These problems have necessitated a change in the traditional concepts and practices of power systems. There are emerging technologies available, which can help electric companies to deal with above problems. One of such technologies is Flexible AC Transmission System (FACTS) [15][16]. As discussed in Chapter 1, within the family of the converter based FACTS, there are a number of FACTS devices available, including the Static Synchronous Compensator (STATCOM) [17], the Static Synchronous Series Compensator (SSSC) [18][19], the Unified Power Flow Controller (UPFC) [20][21], and the latest FACTS devices [22]-[32], etc.

Among the converter based FACTS-devices, STATCOM may be one of the popular FACTS-devices, which has many installations in electric utilities worldwide. Considering the practical applications of the STATCOM in power systems, it is of importance and interest to investigate the possible multi-control functions of the STATCOM as well as model these functions in power system steady state

operation and control, such that the various control capabilities can be fully employed, and the benefits of applications of the STATCOM may be fully realized. Nine multi-control functions of the STATCOM will be presented:

- There are two solutions associated with the current magnitude control function, which are discussed. Alternative formulations of the control function to avoid the multiple solutions of the current magnitude control are proposed. Two reactive power control functions are proposed, which are interesting and attractive, and they can be used in either normal control or security control of deregulated electric power systems.
- Full consideration of the current and voltage operating constraints associated with the STATCOM and their detailed implementation in Newton power flow will be described. Effort is particularly made on the enforcement of simultaneous multiple violated internal and external constraints associated with the STATCOM. A strategy will be presented to deal with the multiple constraints enforcement problem.

2.2.1 Multi-Control Functional Model of STATCOM for Power Flow Analysis

2.2.1.1 Operation Principles of the STATCOM

A STATCOM is usually used to control transmission voltage by reactive power shunt compensation. Typically, a STATCOM consists of a coupling transformer, an inverter and a DC capacitor, which is shown in Fig. 1.11. For such an arrangement, in ideal steady state analysis, it can be assumed that the active power exchange between the AC system and the STATCOM can be neglected, and only the reactive power can be exchanged between them.

2.2.1.2 Power Flow Constraints of the STATCOM

Based on the operating principle of the STATCOM, the equivalent circuit can be derived, which is given in Fig. 2.1. In the derivation, it is assumed that (a) harmonics generated by the STATCOM are neglected; (b) the system as well as the STATCOM are three phase balanced.

Then the STATCOM can be equivalently represented by a controllable fundamental frequency positive sequence voltage source V_{sh} . In principle, the STATCOM output voltage can be regulated such that the reactive power of the STATCOM can be changed.

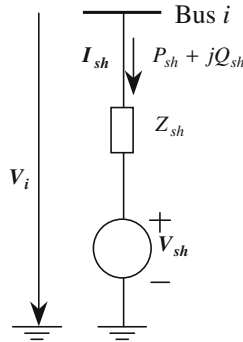


Fig. 2.1. STATCOM equivalent circuit

According to the equivalent circuit of the STATCOM shown in Fig. 2.1, suppose $V_{sh} = V_{sh} \angle \theta_{sh}$, $V_i = V_i \angle \theta_i$, then the power flow constraints of the STATCOM are:

$$P_{sh} = V_i^2 g_{sh} - V_i V_{sh} (g_{sh} \cos(\theta_i - \theta_{sh}) + b_{sh} \sin(\theta_i - \theta_{sh})) \quad (2.2)$$

$$Q_{sh} = -V_i^2 b_{sh} - V_i V_{sh} (g_{sh} \sin(\theta_i - \theta_{sh}) - b_{sh} \cos(\theta_i - \theta_{sh})) \quad (2.3)$$

where $g_{sh} + jb_{sh} = 1/Z_{sh}$.

The operating constraint of the STATCOM is the active power exchange via the DC-link as described by:

$$PE = \text{Re}(V_{sh} I_{sh}^*) = 0 \quad (2.4)$$

where $\text{Re}(V_{sh} I_{sh}^*) = V_{sh}^2 g_{sh} - V_i V_{sh} (g_{sh} \cos(\theta_i - \theta_{sh}) - b_{sh} \sin(\theta_i - \theta_{sh}))$.

2.2.1.3 Multi-Control Functions of the STATCOM

In the practical applications of a STATCOM, it may be used for controlling one of the following parameters [34]:

1. voltage magnitude of the local bus, to which the STATCOM is connected;
2. reactive power injection to the local bus, to which the STATCOM is connected;
3. impedance of the STATCOM;
4. current magnitude of the STATCOM while the current I_{sh} leads the voltage injection V_{sh} by 90° ;
5. current magnitude of the STATCOM, while the current I_{sh} lags the voltage injection V_{sh} by 90° ;
6. voltage injection;
7. voltage magnitude at a remote bus;

8. reactive power flow;
9. apparent power or current control of a local or remote transmission line.

Among these control options, control of the voltage of the local bus, which the STATCOM is connected to, is the most-recognized control function. The other control possibilities have not fully been investigated in power flow analysis. The mathematical descriptions of the control functions are presented as follows.

Control mode 1: Bus voltage control

The bus control constraint is as follows:

$$V_i - V_i^{Spec} = 0 \quad (2.5)$$

where V_i^{Spec} is the bus voltage control reference.

Control mode 2: Reactive power control

In this control mode, the reactive power generated by the STATCOM is controlled to a reactive power injection reference. Mathematically, such a control constraint is described as follows:

$$Q_{sh} - Q_{sh}^{Spec} = 0 \quad (2.6)$$

where Q_{sh}^{Spec} is the specified reactive power injection control reference. Q_{sh} , which is given by (2.3), is the actual reactive power generated by the STATCOM.

Control mode 3: Control of equivalent impedance

In principle, a STATCOM compensation can be equivalently represented by an imaginary impedance or reactance. In this control mode, V_{sh} is regulated to control the equivalent reactance of the STATCOM to a specified reactance reference:

$$X_{shunt} - X_{shunt}^{Spec} = 0 \quad (2.7)$$

where X_{shunt}^{Spec} is the specified reactance control reference of the STATCOM. X_{shunt} is the equivalent reactance of the STATCOM. X_{shunt} , which is a function of the state variables V_i and V_{sh} , is defined as:

$$X_{shunt} = \text{Im}(V_{sh} / I_{sh}) = \text{Im}[V_{sh} Z_{sh} / (V_i - V_{sh})] \quad (2.8)$$

Control mode 4: Control of current magnitude - Capacitive compensation

In this control mode, a STATCOM is used to control the magnitude of the current I_{sh} of the STATCOM to a specified current magnitude control reference. The control constraint may be represented by $I_{sh} - I_{sh}^{Spec} = 0$. However it is found that there are two solutions corresponding to this control constraint. Due to the problem incurred, the power flow solution with such a constraint may arbitrarily converge to one of the two solutions. In section 2.2.4, further analysis is given to show the two solutions associated with this current magnitude control constraint.

In order to avoid the above non-unique solution problem, an alternative formulation of the current magnitude control is introduced here. Since $I_{sh} = I_{sh}^{Spec}$, if further assume I_{sh} leads V_{sh} by 90° , then $I_{sh} = I_{sh}^{Spec} \angle(\theta_{sh} + 90^\circ)$. I_{sh} can also be defined by $I_{sh} = \frac{V_i - V_{sh}}{Z_{sh}}$, then we have:

$$I_{sh}^{Spec} \angle(\theta_{sh} + 90^\circ) = (V_i - V_{sh}) / Z_{sh} \quad (2.9)$$

Mathematically, such a control mode can be described by one of the following equations:

$$\begin{aligned} \text{Re}(I_{sh}^{Spec} \angle(\theta_{sh} + 90^\circ)) &= \text{Re}[(V_i - V_{sh}) / Z_{sh}] \\ \text{or} \quad \text{Im}(I_{sh}^{Spec} \angle(\theta_{sh} + 90^\circ)) &= \text{Im}[(V_i - V_{sh}) / Z_{sh}] \end{aligned} \quad (2.10)$$

The formulation of (2.10) can force the power flow to converge to one of the two solutions. This control mode has a clear physical meaning. Since I_{sh} leads V_{sh} by 90° , this control mode provides capacitive reactive power compensation while keeping the current magnitude constant.

Control mode 5: Control of current magnitude - Inductive compensation

In order to circumvent the same problem mentioned above, new formulation of the current control constraint needs to be introduced. In this control mode, the STATCOM is used to control the magnitude of the current I_{sh} of the STATCOM while I_{sh} lags V_{sh} by 90° . Mathematically, such a control mode may be described by:

$$\begin{aligned} \text{Re}(I_{sh}^{Spec} \angle(\theta_{sh} - 90^\circ)) &= \text{Re}[(V_i - V_{sh}) / Z_{sh}] \\ \text{or} \quad \text{Im}(I_{sh}^{Spec} \angle(\theta_{sh} - 90^\circ)) &= \text{Im}[(V_i - V_{sh}) / Z_{sh}] \end{aligned} \quad (2.11)$$

Similar to that of (2.10), the formulation of (2.11) can force the power flow to converge to the other one of the two possible solutions. This control mode also has a clear physical meaning, that is, it provides inductive reactive power compensation while keeping the current magnitude constant.

Control mode 6: Control of equivalent injected voltage magnitude V_{sh} of STATCOM

In this control mode, a STATCOM is used to control the magnitude of the voltage V_{sh} of the STATCOM to a specified voltage magnitude control reference. The control constraint is as follows:

$$V_{sh} - V_{sh}^{Spec} = 0 \quad (2.12)$$

where V_{sh} is the voltage magnitude of the equivalent injected voltage V_{sh} of the STATCOM. V_{sh}^{Spec} is the voltage control reference.

Control mode 7: Remote voltage magnitude control

In this control mode, the STATCOM is used to control a remote voltage magnitude at bus j to a specified voltage control reference. Mathematically, such a control constraint is described as follows:

$$V_j - V_j^{Spec} = 0 \quad (2.13)$$

where V_j is the voltage magnitude of a remote bus, and V_j^{Spec} is the specified remote bus voltage control reference.

Control mode 8: Local or remote reactive power flow control

In this control mode, the STATCOM is used to control either the local reactive power flow of a transmission line connected to the local bus or the reactive power flow of a remote transmission line to a specified reactive power flow control reference. Mathematically, such a control constraint is described as follows:

$$Q_{jk} - Q_{jk}^{Spec} = 0 \quad (2.14)$$

where Q_{jk} is the reactive power flow leaving bus j on the transmission line $j-k$. Q_{jk}^{Spec} is the reactive power flow control reference.

Control mode 9: Local or remote control of (maximum) apparent power

In this control mode, the STATCOM is used to control either the apparent power of a transmission line connected to the local bus or the apparent power of a remote transmission line to a specified power control reference. Mathematically, such a control constraint is described as follows:

$$S_{jk} - S_{jk}^{Spec} = 0 \quad (2.15)$$

where $S_{jk} = \sqrt{(P_{jk})^2 + (Q_{jk})^2}$ is the apparent power of the transmission line $j-k$ while P_{jk} and Q_{jk} are the active and reactive power of the transmission line. S_{jk}^{Spec} is the apparent power control reference, which may be the power rating of the transmission line.

Alternatively, the current magnitude of the transmission line may be controlled. The constraint can be represented by:

$$I_{jk} - I_{jk}^{Spec} = 0 \quad (2.16)$$

where $I_{jk} = \frac{\sqrt{(P_{jk})^2 + (Q_{jk})^2}}{V_j}$ is the actual current magnitude on the transmission line $j-k$. I_{jk}^{Spec} is the current control reference, which may be the current rating of the transmission line.

Remarks on control modes 8 and 9:

- It is well recognized that a STATCOM may control a local bus voltage. However, it has not been recognized that a STATCOM may be used to control power flow of a transmission line. In addition to the local voltage control mode, it is important to explore other possible applications, such that the capabilities of the STATCOM can be fully employed.
- The control modes 8 and 9 presented in (2.14) and (2.15) or (2.16) introduce possible innovative applications of the STATCOM in power flow control.
- The reactive power flow control mode 8 can be used to control reactive power flow of an adjacent transmission line.
- The apparent power or current control mode 9 can be used to control the apparent power or current of an adjacent transmission line.
- In an electricity market, transmission congestion management by shunt reactive power control resources like STATCOM may be cheaper than by re-dispatching of active generating power. In this situation, control modes 8 and 9 may be very attractive. However, the control modes should not be overestimated. The controls may be very effective when there is excessive reactive power flow on a transmission line.
- Both control modes 8 and 9 of the STATCOM may be used in not only normal control when there is excessive reactive flowing on a transmission line but also security control of electric power systems when there is a violation of the thermal constraint of a transmission line.

Equations (2.5) - (2.7), (2.10) - (2.16) can be generally written as:

$$\Delta F(\mathbf{x}) = F(\mathbf{x}, f^{Spec}) = 0 \quad (2.17)$$

where $\mathbf{x} = [\theta_i, V_i, \theta_j, V_j, \theta_k, V_k, \theta_{sh}, V_{sh}]^t$. f^{Spec} is the control reference.

2.2.1.4 Voltage and Thermal Constraints of the STATCOM

The equivalent voltage injection V_{sh} bound constraints:

$$V_{sh}^{\min} \leq V_{sh} \leq V_{sh}^{\max} \quad (2.18)$$

$$-\pi \leq \theta_{sh} \leq \pi \quad (2.19)$$

where $V_{sh}^{\max}$ is the voltage rating of the STATCOM, while $V_{sh}^{\min}$ is the minimal voltage limit of the STATCOM.

The current flowing through a STATCOM should be less than its current rating:

$$I_{sh} \leq I_{sh}^{\max} \quad (2.20)$$

where $I_{sh}^{\max}$ is current rating of the STATCOM converter while I_{sh} is the magnitude of the current through the STATCOM and given by:

$$I_{sh} = |(V_i - V_{sh}) / Z_{sh}| = \sqrt{V_i^2 + V_{sh}^2 - 2V_i V_{sh} \cos(\theta_i - \theta_{sh})} / |Z_{sh}| \quad (2.21)$$

The constraints (2.18) and (2.20) are the internal constraints of the STATCOM.

2.2.1.5 External Voltage Constraints

In the practical operation of power systems, normally, a bus voltage should be within its operating limits. For all the control modes except the typical control mode 1, the voltage of bus i , to which the STATCOM is connected, should be constrained by:

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (2.22)$$

For all the control modes except the control mode 7, the voltage of the remote bus j may be monitored. The operating constraints of the voltage may be described by:

$$V_j^{\min} \leq V_j \leq V_j^{\max} \quad (2.23)$$

where $V_j^{\max}$ and $V_j^{\min}$ are the specified maximal and minimal voltage limits, respectively, at the remote bus j .

It should be pointed out, that other types of external limits other than (2.22), (2.23) may also be included.

2.2.2 Implementation of Multi-Control Functional Model of STATCOM in Newton Power Flow

2.2.2.1 Multi-Control Functional Model of STATCOM in Newton Power Flow

A STATCOM has only one degree of freedom for control since the active power exchange with the DC link should be zero at any time. The STATCOM may be used to control one of the nine parameters. The Newton power flow equation including power mismatch constraints of buses i, j, k , and the STATCOM control constraints may be represented by:

$$\begin{bmatrix}
\frac{\partial PE}{\partial \theta_{sh}} & \frac{\partial PE}{\partial V_{sh}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & 0 & 0 & 0 & 0 \\
\frac{\partial F}{\partial \theta_{sh}} & \frac{\partial F}{\partial V_{sh}} & \frac{\partial F}{\partial \theta_i} & \frac{\partial F}{\partial V_i} & \frac{\partial F}{\partial \theta_j} & \frac{\partial F}{\partial V_j} & \frac{\partial F}{\partial \theta_k} & \frac{\partial F}{\partial V_k} \\
\frac{\partial P_i}{\partial \theta_{sh}} & \frac{\partial P_i}{\partial V_{sh}} & \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_j} & \frac{\partial P_i}{\partial \theta_k} & \frac{\partial P_i}{\partial V_k} \\
\frac{\partial Q_i}{\partial \theta_{sh}} & \frac{\partial Q_i}{\partial V_{sh}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} & \frac{\partial Q_i}{\partial \theta_k} & \frac{\partial Q_i}{\partial V_k} \\
0 & 0 & \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_j} & \frac{\partial P_j}{\partial \theta_k} & \frac{\partial P_j}{\partial V_k} \\
0 & 0 & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j} & \frac{\partial Q_j}{\partial \theta_k} & \frac{\partial Q_j}{\partial V_k} \\
0 & 0 & \frac{\partial P_k}{\partial \theta_i} & \frac{\partial P_k}{\partial V_i} & \frac{\partial P_k}{\partial \theta_j} & \frac{\partial P_k}{\partial V_j} & \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial V_k} \\
0 & 0 & \frac{\partial Q_k}{\partial \theta_i} & \frac{\partial Q_k}{\partial V_i} & \frac{\partial Q_k}{\partial \theta_j} & \frac{\partial Q_k}{\partial V_j} & \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial V_k}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_{sh} \\
\Delta V_{sh} \\
\Delta \theta_i \\
\Delta V_i \\
\Delta \theta_j \\
\Delta V_j \\
\Delta \theta_k \\
\Delta V_k
\end{bmatrix}
=
\begin{bmatrix}
-PE \\
-\Delta F \\
-\Delta P_i \\
-\Delta Q_i \\
-\Delta P_j \\
-\Delta Q_j \\
-\Delta P_k \\
-\Delta Q_k
\end{bmatrix} \quad (2.24)$$

where ΔP_l and ΔQ_l ($l = i, j, k$) are, respectively, the real and reactive power mismatches at bus l .

The STATCOM has two state variables θ_{sh} and V_{sh} , and two equalities. The two equalities formulate the first two rows of the above Newton equation. The first equality is the active power balance equation described by (2.4), while the second equality is the control constraint of the STATCOM, which is generally described by (2.17).

2.2.2.2 Modeling of Constraint Enforcement in Newton Power Flow

If the injected voltage V_{sh} violates its voltage limit either $V_{sh}^{\max}$ or $V_{sh}^{\min}$, V_{sh} is simply kept at the limit. Mathematically, the following equality should hold:

$$\begin{aligned}
V_{sh} - V_{sh}^{\max} &= 0, & \text{if } V_{sh} \geq V_{sh}^{\max} \\
V_{sh} - V_{sh}^{\min} &= 0, & \text{if } V_{sh} \leq V_{sh}^{\min}
\end{aligned} \quad (2.25)$$

In the meantime, the control constraint of (2.17) should be released. Similarly, the violations of the other constraints such as (2.20), (2.22), and (2.23) can be handled. The principle here is that when an inequality constraint is violated, it becomes an equality being kept at its limit while releasing the control constraint (2.17).

Due to the fact that the STATCOM has only one control degree of freedom, it is assumed that each time only one inequality constraint is violated. Similar to (2.25), the general constraint enforcement equation of (2.18), (2.20), (2.22), and (2.23) may be written as:

$$\Delta G(\mathbf{x}) = G(\mathbf{x}) - G^{Spec} = 0 \quad (2.26)$$

where $\mathbf{x} = [\theta_{sh}, V_{sh}, \theta_i, V_i, \theta_j, V_j, \theta_k, V_k]^t$. G^{Spec} is the limit of the internal voltage or current of the STATCOM or the limit of the external bus voltage.

When any of the inequalities in (2.18), (2.20), (2.22), and (2.23) is violated and enforced, in the Newton power flow equation of (2.24), the control constraint (2.17) is replaced by (2.26).

The constraint enforcement only affects the second row of the Newton equation while other elements are unchanged. However, if two or more inequality constraints associated with a STATCOM are violated simultaneously, the constraint enforcement will become very complex. A strategy will be presented in section 2.2.3. In power flow calculations, a special initialization of the STATCOM is not needed.

2.2.3 Multi-Violated Constraints Enforcement

2.2.3.1 Problem of Multi-Violated Constraints Enforcement

Basically, there are internal and external inequality constraints that may need to be considered for the operation of a STATCOM. The practical operation of a STATCOM is primarily constrained by its two internal operation inequalities, i.e., its voltage and thermal constraints given by (2.18) and (2.20), respectively. In the meantime, a STATCOM should also be able to monitor and control the local voltage at bus i , and the remote voltage at bus j within their limits. In other words, the two external voltage constraints given by (2.22), (2.23) should be satisfied for some operating modes while a STATCOM's internal constraints are not violated.

When any one of the inequality constraints is violated, it should be enforced while the associated control constraint described by (2.17) needs to be released. As pointed out in the previous section, in principle, the STATCOM is only able to enforce one of the inequalities each time since it has only one control degree of freedom. Thus, difficulty will appear if two or more internal or external constraints of a STATCOM are violated at the same time.

2.2.3.2 Concepts of Dominant Constraint and Dependent Constraint

Suppose there are two constraints, say constraint A and B, associated with a STATCOM. Assume the two constraints are violated simultaneously, if after the constraint A is enforced, the violation of the constraint B is automatically resolved. In this case, constraint A is called a dominant constraint, and constraint B is called a dependent constraint. The concept of dominant and dependent constraints is applicable to situations when there are more than two violated constraints.

In the following, a strategy for enforcement of two or more simultaneous violated constraints will be discussed based on the concepts of the dominant con-

straint and dependent constraint.

2.2.3.3 Strategy for Multi-Violated Constraints Enforcement

Generally, an internal constraint has priority to be enforced if both the internal and external constraints are violated simultaneously. When multiple constraints associated with the STATCOM are violated, a strategy is proposed as follows:

Step 0: Formulate violated constraints set of STATCOM

- (a) after the K^{th} power flow iterations, formulate the violated constraints set of the STATCOM.
- (b) find the priority order of the violated constraints.
- (c) suppose that there are n violated constraints, and they are in the priority order of $A_1, A_2, \dots, A_n$.

Step 1: Identification of dominant constraint

$KK = K$

Loop $i = 1, n$

- (a) At KK^{th} power flow iteration, constraint A_i is enforced
- (b) After the iteration, check whether the other violated constraints are within their limits. If yes, constraint A_i is the dominant constraint, go to Step 3.
- (c) Set power flow iteration count $KK = KK + 1$

End loop

Step 2: Dominant constraint not found

- (a) If there are violated internal constraints, choose one of them to be enforced until the power flow converges. For the violated internal constraint chosen to be enforced, it may be the dominant constraint of all the violated internal constraint.
- (b) Otherwise choose one of the external constraints to be enforced until the power flow converges.

Step 3: Dominant constraint found

The dominant constraint out of all the violated constraints is found, the constraint is to be continuously enforced until the power flow converges.

The above constraint enforcement algorithm embedded in the Newton power flow calculations may be introduced after the power flow moderately converges, say after one or two Newton power flow iterations.

2.2.4 Multiple Solutions of STATCOM with Current Magnitude Control

For the STATCOM shown in Fig. 2.1, the following active power constraint, which represents the active power exchange between the AC system and the STATCOM, should be held at any instant:

$$\text{Re}(V_{sh} I_{sh}^*) = V_{sh}^2 g_{sh} - V_i V_{sh} (g_{sh} \cos(\theta_i - \theta_{sh}) - b_{sh} \sin(\theta_i - \theta_{sh})) = 0 \quad (2.27)$$

In order to simplify the derivation, assume the resistance of the STATCOM coupling transformer is neglected, the above equation becomes:

$$\text{Re}(V_{sh} I_{sh}^*) = V_i V_{sh} b_{sh} \sin(\theta_i - \theta_{sh}) = 0 \quad (2.28)$$

Since $V_i \neq 0$ and $V_{sh} \neq 0$, from the above equation, we have:

$$\theta_i - \theta_{sh} = 0 \text{ or } \theta_i - \theta_{sh} = \pi \quad (2.29)$$

The conventional current magnitude control is:

$$I_{sh} - I_{sh}^{Spec} = 0 \quad (2.30)$$

Substitute (2.21) into the above equation, we have:

$$I_{sh}^{Spec} |Z_{sh}| = \sqrt{V_i^2 + V_{sh}^2 - 2V_i V_{sh} \cos(\theta_i - \theta_{sh})} \quad (2.31)$$

If $\theta_i - \theta_{sh} = 0$, the above equation becomes:

$$I_{sh}^{Spec} |Z_{sh}| = \sqrt{V_i^2 + V_{sh}^2 - 2V_i V_{sh}} = \pm(V_i - V_{sh}) \quad (2.32)$$

The above equation can be written as (2.33) showing the two solutions to the current magnitude control given by (2.30):

$$V_{sh} = V_i \pm I_{sh}^{Spec} |Z_{sh}| \quad (2.33)$$

If $\theta_i - \theta_{sh} = \pi$, we have:

$$I_{sh}^{Spec} |Z_{sh}| = \sqrt{V_i^2 + V_{sh}^2 + 2V_i V_{sh}} = V_i + V_{sh} \quad (2.34)$$

Then we have:

$$V_{sh} = I_{sh}^{Spec} |Z_{sh}| - V_i \quad (2.35)$$

Note that $I_{sh}^{Spec} |Z_{sh}| \ll V_i$, thus $V_{sh} = I_{sh}^{Spec} |Z_{sh}| - V_i < 0$. This is not a feasible solution of (2.30). Therefore there are only two solutions to (2.30), which are given by (2.33).

2.2.5 Numerical Examples

2.2.5.1 Multi-Control Capabilities of STATCOM

To verify the STATCOM model and explore the multi-control capabilities of the STATCOM, numerical studies have been carried out on the IEEE 30-bus system, IEEE 118-bus system and IEEE 300-bus system. In the tests, a convergence tolerance of $1.0\text{e-}12$ p.u. (or $1.0\text{e-}10$ MW/MVar) is used for maximal absolute bus power mismatches and power flow control mismatches. The single-line circuit diagram of the IEEE 30-bus system is shown in Fig. 2.2.

In order to show the multi-control capabilities of the STATCOM in power flow studies, cases 1-10 on the IEEE 30-bus system have been carried out. Case 1 is the base case without STATCOM. In cases 2-10, a STATCOM is installed at bus 12. In cases 2-10, nine different control modes of the STATCOM have been simulated. Control references for each control mode and corresponding number of iterations are shown in column 3 and column 4, respectively, in Table 2.1.

Power flow solutions of the STATCOM state variables for case 5 and case 6 are shown in Table 2.2. The two cases are corresponding to two constant current control modes of the STATCOM, respectively. However, if the current magnitude control in (2.30) is applied, the STATCOM solution may converge arbitrarily to one of the above two solutions of case 5 and case 6.

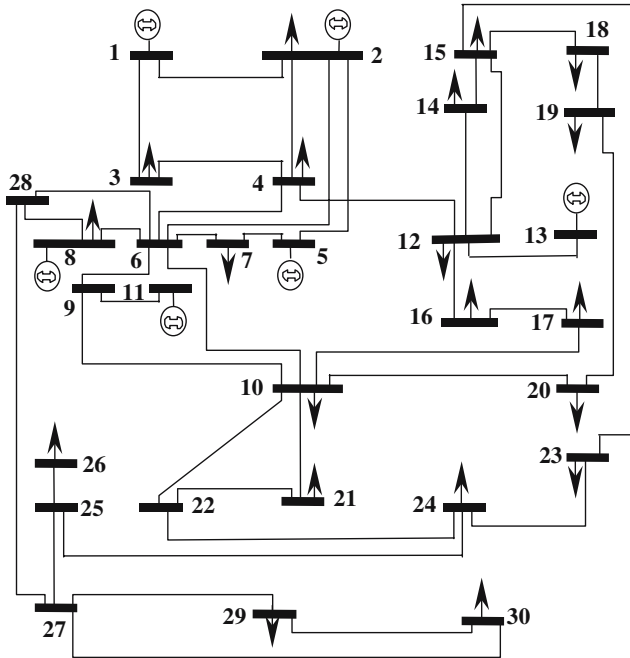


Fig. 2.2. Single-line circuit diagram of the IEEE 30-bus system.

Table 2.1. Results of STATCOM multi-control on the IEEE 30-bus system

Case No.	Control mode	Control reference in p.u.	Iterations
1	None	None	4
2	1	$V_{12}^{Spec} = 1.0$	4
3	2	$Q_{sh}^{Spec} = 1.0$	4
4	3	$X_{shunt}^{Spec} = -10.0$	4
5	4	$I_{sh}^{Spec} = 0.3$	4
6	5	$I_{sh}^{Spec} = 0.3$	4
7	6	$V_{sh}^{Spec} = 1.0$	4
8	7	$V_{17}^{Spec} = 1.0$	4
9	8	$Q_{13,12}^{Spec} = 0.0$	4
10	9	$S_{9,11}^{Spec} = 0.22$	5

The power flows of line 13-12 and line 9-11 of case 1, case 9 and case 10 are given by Table 2.3. In comparison to the power flows of line 13-12 of case 1 and case 9, the STATCOM of case 9 is able to control the reactive power flow of line 13-12 to the specified control reference 0.0 p.u., while the active power flow is almost unchanged. By driving the reactive power flow on the line to zero using STATCOM, the un-used (available) transmission line capacity can be increased. It can be seen that the base case reactive power of line 13-12 is 0.384 p.u., so the reactive power flow control by the STATCOM is significant.

Comparing the power flows of line 9-11 in case 1 and case 10, it can be found that in case 10, the apparent power of the remote line 9-11 can be controlled to the specified control reference of 0.22 p.u., while the active power is almost unchanged.

Table 2.2. Results of case 5 and case 6 for the IEEE 30-bus system

Case No.	Power flow solution of V_{sh} (in p.u.), θ_{sh} (in degree)
5	$V_{sh} = 0.9831$, $\theta_{sh} = -9.3^\circ$, Inductive compensation
6	$V_{sh} = 1.0449$, $\theta_{sh} = -9.6^\circ$, Capacitive compensation

Table 2.3. Power flows of transmission line of case 1, case 9 and case 10 for the IEEE 30-bus System

Case No.	Power flows of line 13-12	Power flow of 9-11
1	$P_{13,12} = 0.169$, $Q_{13,12} = \mathbf{0.384}$, $S_{13,12} = 0.420$	$P_{9,11} = -0.179$, $Q_{9,11} = -0.288$, $S_{9,11} = \mathbf{0.420}$
9	$P_{13,12} = 0.169$, $Q_{13,12} = \mathbf{0.000}$, $S_{13,12} = 0.169$	—
10	—	$P_{9,11} = -0.179$, $Q_{9,11} = -0.127$, $S_{9,11} = \mathbf{0.220}$

The control reference is much lower than the apparent power of 0.42 p.u. of line 9-11 in case 1 (base case). The control mode 9 may be used when the thermal limit of a transmission line is violated or the un-used transmission capacity needs to be increased.

Cases 9 and 10 reveal that the STATCOM has very little influence on the active power flow of a transmission line, while it has strong capability of controlling reactive power on a transmission line. In addition, both control modes 8 and 9 of a STATCOM can be used for local control of reactive power flow on a transmission line. The control modes may be attractive when, in electricity market environments, re-dispatching active power becomes much more expensive than controlling reactive power.

Test results on the IEEE 118-bus system and the IEEE-300 bus system can be found in [34].

2.2.5.2 Multi-Violated STATCOM Constraints Enforcement

The following case is to show the enforcement of a single constraint violation of a STATCOM, which is as follows:

Case 12: This is similar to case 2, but assume that a current limit of $I_{sh}^{\max} = 0.9\text{p.u.}$ is applied to the STATCOM.

The power flow algorithm converges in 4 iterations. In the tests, it has been found that for single constraint violation of the STATCOM based on the IEEE 30, 118, 300 bus systems, the power flow algorithm can converge in the same number of iterations as that of base case power flow solution. Occasionally, the power flow algorithm needs one or more extra iterations.

The following case is used to illustrate the enforcement of the multiple violated voltage and current constraints associated the STATCOM on the IEEE 30 bus system.

Case 13: This is similar to case 3. In this case, it is assumed that the two internal constraints and two external voltage constraints at bus 12 and 17 are violated when the following voltage and current limits are applied:

$$V_{sh}^{\max} = 1.065 \text{ p.u.}, I_{sh}^{\max} = 0.46 \text{ p.u.}, V_{12}^{\max} = 1.05 \text{ p.u.}, V_{17}^{\max} = 1.00 \text{ p.u.}.$$

The detailed enforcement of the four violated constraints is presented in Table 2.4. In the enforcement of the multiple violated constraints, much effort has been made on the identification of the dominant constraint. Case 13 shows the worst scenario, in which the algorithm took four power flow iterations to identify the dominant constraint. However, if another constraint, for instance V_{sh} rather than the external remote voltage constraint were the dominant constraint, the algorithm would complete the identification in one power flow iteration.

Table 2.4. Enforcement of multi-violated constraints of a STATCOM in the IEEE 30-bus system

Iteration count	Enforcement of the multi-violated constraints
0	None
1	Form the violated constraint set $\{V_{sh}, I_{sh}, V_{12}, V_{17}\}$ and priority order
2	V_{sh} is enforced, it is not the dominant constraint
3	I_{sh} is enforced, it is not the dominant constraint. But it is the dominant constraint between V_{sh} and I_{sh}
4	V_{12} is enforced, it is not the dominant constraint
5	V_{17} is enforced, it is the dominant constraint. Identification of dominant and dependent constraints pair complete
6	V_{17} is enforced
7	The power flow converges

The multi-violated STATCOM constraints enforcement algorithm has successfully solved further cases on the IEEE 118-bus system and IEEE 300-bus system [34].

This section has defined a multi-control functional model for STATCOM suitable for power system steady state operational studies. Nine control modes have been incorporated into the STATCOM model. There are two solutions associated with the current magnitude control. Alternative formulations of the control mode are introduced. The reactive power flow and apparent power control modes, which may be used in either normal control or security control of electric power systems, are interesting and attractive when, in electricity market environments, redispatching active power becomes expensive, and there is excessive reactive

power on a transmission line. Numerical results based on the IEEE 30-bus system, IEEE 118-bus system, and IEEE 300-bus system with the multi-control functional STATCOM have demonstrated the feasibility and effectiveness of the proposed multi-control functional STATCOM model for power system steady state operation and control.

Furthermore, a comprehensive strategy for enforcement of the multi-violated STATCOM internal and external constraints has been described. Numerical results show that the strategy proposed can enforce successfully the multiple violated voltage and current (inequality) constraints associated with a STATCOM on the IEEE systems. The constraint enforcement strategy for multiple violated constraints associated with a STATCOM may be further enhanced by incorporation of advanced expert rules into the algorithm. The key issue is to reduce the effort for identifying the dominant constraint.

2.3 Modeling of Multi-Control Functional SSSC

It is found that, in the past, much effort has been paid in the modeling of the UPFC for power flow and optimal power flow analysis, while few work has been published on the modeling of the SSSC, in particular, for power flow analysis. In this section, multi-control modes of the SSSC will be discussed, in particular:

- The multi-control modes of the SSSC will be explored. A multi-control functional model of the SSSC, which can be used for steady state controlling any of the following parameters, (a) the active power flow of the transmission line, (b) the reactive power flow the transmission line, (c) the bus voltage, and (d) the impedance of the transmission line, will be presented.
- Fully consideration of the current and voltage operating constraints of the SSSC and detailed direct implementation of these in Newton power flow will be described.

2.3.1 Multi-Control Functional Model of SSSC for Power Flow Analysis

2.3.1.1 Operation Principles of the SSSC

A SSSC [18][19] usually consists of a coupling transformer, an inverter and a capacitor. As shown in Fig. 2.3, the SSSC is series connected with a transmission line through the coupling transformer.

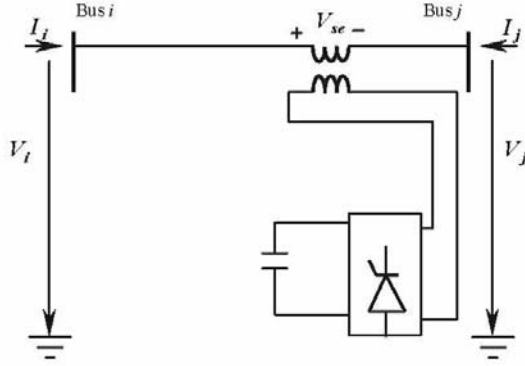


Fig. 2.3. SSSC operation principles

It is assumed here that the transmission line is series connected via the SSSC bus j . The active and reactive power flows of the SSSC branch i - j entering the bus j are equal to the sending end active and reactive power flows of the transmission line, respectively. In principle, the SSSC can generate and insert a series voltage, which can be regulated to change the impedance (more precisely reactance) of the transmission line. In this way, the power flow of the transmission line or the voltage of the bus, which the SSSC is connected with, can be controlled.

2.3.1.2 Equivalent Circuit and Power Flow Constraints of SSSC

An equivalent circuit of the SSSC as shown in Fig. 2.4 can be derived based on the operation principle of the SSSC. In the equivalent, the SSSC is represented by a voltage source V_{se} in series with a transformer impedance. In the practical operation of the SSSC, V_{se} can be regulated to control the power flow of line i - j or the voltage at bus i or j .

In the equivalent circuit, $V_{se} = V_{se} \angle \theta_{se}$, $V_i = V_i \angle \theta_i$, $V_j = V_j \angle \theta_j$, then the bus power flow constraints of the SSSC are:

$$P_{ij} = V_i^2 g_{ii} - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) - V_i V_{se} (g_{ij} \cos(\theta_i - \theta_{se}) + b_{ij} \sin(\theta_i - \theta_{se})) \quad (2.36)$$

$$Q_{ij} = -V_i^2 b_{ii} - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) - V_i V_{se} (g_{ij} \sin(\theta_i - \theta_{se}) - b_{ij} \cos(\theta_i - \theta_{se})) \quad (2.37)$$

$$P_{ji} = V_j^2 g_{jj} - V_i V_j (g_{ij} \cos \theta_{ji} + b_{ij} \sin \theta_{ji}) + V_j V_{se} (g_{ij} \cos(\theta_j - \theta_{se}) + b_{ij} \sin(\theta_j - \theta_{se})) \quad (2.38)$$

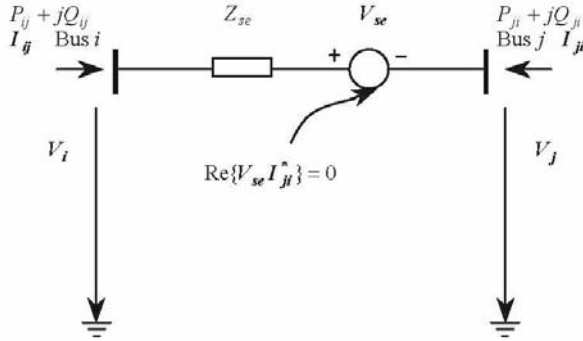


Fig. 2.4. SSSC equivalent circuit

$$\begin{aligned} Q_{ji} = & -V_j^2 b_{jj} - V_i V_j (g_{ij} \sin \theta_{ji} - b_{ij} \cos \theta_{ji}) \\ & + V_j V_{se} (g_{ij} \sin(\theta_j - \theta_{se}) - b_{ij} \cos(\theta_j - \theta_{se})) \end{aligned} \quad (2.39)$$

where $g_{ij} + jb_{ij} = 1/Z_{se}$, $g_{ii} = g_{ij}$, $b_{ii} = b_{ij}$, $g_{jj} = g_{ij}$, $b_{jj} = b_{ij}$.

The operating constraint of the SSSC (active power exchange via the DC link) is:

$$PE = \text{Re}(V_{se} I_{ji}^*) = 0 \quad (2.40)$$

where
$$\begin{aligned} \text{Re}(V_{se} I_{ji}^*) = & -V_i V_{se} (g_{ij} \cos(\theta_i - \theta_{se}) - b_{ij} \sin(\theta_i - \theta_{se})) \\ & + V_j V_{se} (g_{ij} \cos(\theta_j - \theta_{se}) - b_{ij} \sin(\theta_j - \theta_{se})) \end{aligned}$$

2.3.1.3 Multi-Control Functions and Constraints of SSSC

In the practical applications of the SSSC, it may be used for control of any of the following parameters, (a) the active power flow of the transmission line, (b) the reactive power flow of the transmission line, (c) the bus voltage, and (d) the impedance of the transmission line. Therefore, the SSSC may have four control modes. Among the four control modes, the active power flow control mode has been well recognized. The mathematical descriptions of the four control functions of the SSSC are presented as follows.

Control mode 1: Active power flow control

The active power flow control constraint is as follows:

$$P_{ji} - P_{ji}^{Spec} = 0 \quad (2.41)$$

where P_{ji}^{Spec} is the specified active power flow control reference.

Control mode 2: Reactive power flow control

In this function, the reactive power flow control constraint is as follows:

$$Q_{ji} - Q_{ji}^{Spec} = 0 \quad (2.42)$$

where Q_{ji}^{Spec} is the specified reactive power flow control reference. As mentioned, P_{ji} , Q_{ji} are the SSSC branch active and reactive power flows, respectively, leaving the SSSC bus j while the sending end active and reactive power flows of the transmission line are $-P_{ji}$ and $-Q_{ji}$, respectively.

Control mode 3: Bus voltage control

The bus voltage control constraint is given by:

$$\begin{aligned} V_i - V_i^{Spec} &= 0 \\ \text{or } V_j - V_j^{Spec} &= 0 \end{aligned} \quad (2.43)$$

where V_i^{Spec} and V_j^{Spec} are the bus voltage control references.

Control mode 4: Impedance (reactance) control

In this function, V_{se} is regulated to control equivalent reactance of the SSSC to a specified reactance reference:

$$X_{comp} - X_{comp}^{Spec} = 0 \quad (2.44)$$

where X_{comp}^{Spec} is the specified reactance reference. While X_{comp} is a function of the state variables V_i , V_j and V_{se} .

Theoretically, the control mode 4 is equivalent to replacing the entire SSSC with a fixed reactance. However, the problem of modeling of control mode 4 of the SSSC by a fixed reactance is that it would be very difficult to deal with the voltage and current constraints of the SSSC if not impossible. Subsequently, this would cause the change of the structure and dimension of the Newton Jacobian matrix, and increase the complexity of the code. In contrast, the present reactance control mode of the SSSC in (2.44) has no such limitations. Further advantages of the present formulation of the control modes of the SSSC will be discussed in section 2.3.2.

Equations (2.41)-(2.44) can be generally written as:

$$\Delta F(\mathbf{x}) = F(\mathbf{x}) - F^{Spec} = 0 \quad (2.45)$$

where $\mathbf{x} = [\theta_i, V_i, \theta_j, V_j, \theta_{se}, V_{se}]^t$.

2.3.1.4 Voltage and Current Constraints of the SSSC

The equivalent voltage injection V_{se} bound constraints are as follows:

$$0 \leq V_{se} \leq V_{se}^{\max} \quad (2.46)$$

$$-\pi \leq \theta_{se} \leq \pi \quad (2.47)$$

where $V_{se}^{\max}$ is the voltage rating of V_{se} , which may be constant, or may change slightly with changes in the DC bus voltage, depending on the inverter design. In principle, θ_{se} can be any real. Therefore (2.47) is not a real constraint while (2.46) is a real constraint, which should be hold at any time.

The current through each series converter should be within its current rating:

$$I_{se} \leq I_{se}^{\max} \quad (2.48)$$

where $I_{se}^{\max}$ is the current rating of the series converter.

Note the fact that $I_{se} = I_{se} \angle \theta_{se} = \frac{V_i - V_{se} - V_j}{Z_{se}}$, the actual current magnitude through the SSSC can be obtained:

$$I_{se} = I_{ji} \quad (2.49)$$

$$= \sqrt{V_i^2 + V_{se}^2 + V_j^2 - 2V_i V_{se} \cos(\theta_i - \theta_{se}) + 2V_j V_{se} \cos(\theta_j - \theta_{se}) - 2V_i V_j \cos(\theta_i - \theta_j)} / |Z_{se}|$$

2.3.2 Implementation of Multi-Control Functional Model of SSSC in Newton Power Flow

2.3.2.1 Multi-Control Functional Model of SSSC in Newton Power Flow

For the SSSC, the power mismatches, at its buses i, j , respectively, should be held,

$$\Delta P_i = P_{g_i} - P_{d_i} - P_i = 0 \quad (2.50)$$

$$\Delta Q_i = Q_{g_i} - Q_{d_i} - Q_i = 0 \quad (2.51)$$

$$\Delta P_j = P_{g_j} - P_{d_j} - P_j = 0 \quad (2.52)$$

$$\Delta Q_j = Q_{g_j} - Q_{d_j} - Q_j = 0 \quad (2.53)$$

where P_k, Q_k are, respectively, the real and reactive power leaving the bus k ($k=i, j, \dots$). These are sum of the real and reactive power flows including those given by (2.50)-(2.53), respectively. While P_{g_k}, Q_{g_k} are, respectively, the real and reactive generating power entering the bus k , and P_{d_k}, Q_{d_k} are, respectively, the real and reactive load leaving the bus k .

For the SSSC, it has only one control degree of freedom since the active power exchange with the DC link should be zero at any time. So the SSSC may be used to control only one of the following parameters, (a) the active power flow on the transmission line, (b) the reactive power flow on the transmission line, (c) the bus voltage, and (d) the impedance (precisely reactance) of the transmission line. A Newton power flow algorithm with simultaneous solution of power flow constraints and power flow control constraints of the SSSC may be represented by (2.54) as follows:

$$\begin{bmatrix}
 \frac{\partial F}{\partial \theta_{se}} & \frac{\partial F}{\partial V_{se}} & \frac{\partial F}{\partial \theta_i} & \frac{\partial F}{\partial V_i} & \frac{\partial F}{\partial \theta_j} & \frac{\partial F}{\partial V_j} \\
 \frac{\partial PE}{\partial \theta_{se}} & \frac{\partial PE}{\partial V_{se}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & \frac{\partial PE}{\partial \theta_j} & \frac{\partial PE}{\partial V_j} \\
 \frac{\partial P_i}{\partial \theta_{se}} & \frac{\partial P_i}{\partial V_{se}} & \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_j} \\
 \frac{\partial Q_i}{\partial \theta_{se}} & \frac{\partial Q_i}{\partial V_{se}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} \\
 \frac{\partial P_j}{\partial \theta_{se}} & \frac{\partial P_j}{\partial V_{se}} & \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_j} \\
 \frac{\partial Q_j}{\partial \theta_{se}} & \frac{\partial Q_j}{\partial V_{se}} & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j}
 \end{bmatrix}
 \begin{bmatrix}
 \Delta \theta_{se} \\
 \Delta V_{se} \\
 \Delta \theta_i \\
 \Delta V_i \\
 \Delta \theta_j \\
 \Delta V_j
 \end{bmatrix}
 =
 \begin{bmatrix}
 -\Delta F \\
 -PE \\
 -\Delta P_i \\
 -\Delta Q_i \\
 -\Delta P_j \\
 -\Delta Q_j
 \end{bmatrix} \quad (2.54)$$

In (2.54) the system Jacobian matrix is split into four blocks by the dotted line. The bottom diagonal block has the same structure as that of the system Jacobian matrix of conventional power flow. Though the terms of the former should consider the contributions from the SSSC. The other three blocks of the system Jacobian matrix in (2.54) are SSSC related.

2.3.2.2 Enforcement of Voltage and Current Constraints for SSSC

As discussed in Section 2.2.3, the basic constraint enforcement strategy is that, when there is an inequality constraint such as the current or voltage inequality constraint of the SSSC is violated, the constraint is enforced by being kept at its limit, while the control equality constraint of the SSSC given by one of the equality constraints (2.41)-(2.44) is released. The constraint enforcement equations of (2.46) and (2.48) can be generalized as:

$$\Delta G(\mathbf{x}) = G(\mathbf{x}) - G^{Spec} = 0 \quad (2.55)$$

where $\mathbf{x} = [\theta_i, V_i, \theta_j, V_j, \theta_{se}, V_{se}]^T$. $G(\mathbf{x}) = V_{se}$ and $G^{Spec} = V_{se}^{\max}$, or $G(\mathbf{x}) = I_{se}$ and $G^{Spec} = I_{se}^{\max}$.

Then when either the voltage constraint (2.46) or the current constraint (2.48) is violated, the Newton power flow equation becomes:

$$\begin{bmatrix}
\frac{\partial G}{\partial \theta_{se}} & \frac{\partial G}{\partial V_{se}} & \frac{\partial G}{\partial \theta_i} & \frac{\partial G}{\partial V_i} & \frac{\partial G}{\partial \theta_j} & \frac{\partial G}{\partial V_j} \\
\frac{\partial PE}{\partial \theta_{se}} & \frac{\partial PE}{\partial V_{se}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & \frac{\partial PE}{\partial \theta_j} & \frac{\partial PE}{\partial V_j} \\
\frac{\partial \theta_{se}}{\partial P_i} & \frac{\partial \theta_{se}}{\partial P_i} & \frac{\partial \theta_{se}}{\partial P_i} & \frac{\partial \theta_{se}}{\partial P_i} & \frac{\partial \theta_{se}}{\partial P_i} & \frac{\partial \theta_{se}}{\partial P_i} \\
\frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_j} & \frac{\partial \theta_{se}}{\partial V_j} \\
\frac{\partial Q_i}{\partial \theta_{se}} & \frac{\partial Q_i}{\partial V_{se}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} \\
\frac{\partial \theta_{se}}{\partial P_j} & \frac{\partial \theta_{se}}{\partial P_j} & \frac{\partial \theta_{se}}{\partial P_j} & \frac{\partial \theta_{se}}{\partial P_j} & \frac{\partial \theta_{se}}{\partial P_j} & \frac{\partial \theta_{se}}{\partial P_j} \\
\frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_j} & \frac{\partial \theta_{se}}{\partial V_j} \\
\frac{\partial Q_j}{\partial \theta_{se}} & \frac{\partial Q_j}{\partial V_{se}} & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j} \\
\frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_{se}} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_i} & \frac{\partial \theta_{se}}{\partial V_j} & \frac{\partial \theta_{se}}{\partial V_j}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_{se} \\
\Delta V_{se} \\
\Delta \theta_i \\
\Delta V_i \\
\Delta \theta_j \\
\Delta V_j
\end{bmatrix}
=
\begin{bmatrix}
-\Delta G \\
-PE \\
-\Delta P_i \\
-\Delta Q_i \\
-\Delta P_j \\
-\Delta Q_j
\end{bmatrix} \quad (2.56)$$

It can be seen, that the formulation of Newton power flow with the constraint enforcement in (2.56) has exactly the same structure as that of Newton power flow without the constraint enforcement in (2.54). This property makes the implementation of the power flow algorithm easy and efficient.

2.3.2.3 Initialization of SSSC in Newton Power Flow

Basically, unlike that for the UPFC, there are no analytical solutions available, which can be used to initialize the values of the SSSC voltage variables in power flow analysis.

In the present implementation, for control modes 1, 2 and 3, the initial values of the voltage angle and magnitude of a SSSC may be set as follows:

$$\theta_{se}^0 = -\frac{\pi}{2} \quad (2.57)$$

$$V_{se}^0 = 0.1 \quad (2.58)$$

and for control mode 4, the initial values of the voltage angle and magnitude of a SSSC are set as follows:

$$\theta_{se}^0 = \begin{cases} \frac{\pi}{2}, & \text{if } X_{comp}^{Spec} > 0 \\ -\frac{\pi}{2}, & \text{if } X_{comp}^{Spec} < 0 \end{cases} \quad (2.59)$$

$$V_{se}^0 = |X_{comp}^{Spec}| \quad (2.60)$$

2.3.3 Numerical Results

Numerical results are carried out on the IEEE 30-bus system, IEEE118-bus system and IEEE 300-bus system. In the test, a convergence tolerance of $1.0\text{e-}12$ p.u. (or $1.0\text{e-}10$ MW/MVar) for maximal absolute bus power mismatches and power flow control mismatches is utilized.

2.3.3.1 Power Flow, Voltage and Reactance Control by the SSSC

In order to show the multi-control capabilities of the SSSC model and performance of the Newton power flow algorithm, the following cases based on the IEEE 30 bus system are carried out.

Case 1: This is a base case IEEE 30 bus system.

Case 2: This is similar to case 1 except that there is a SSSC installed for control of the active power flow of line 12-15. The active power flow control reference is set to $P_{15,12}^{Spec} = -30$ MW, which is more than 60% of its corresponding base case active power flow.

Case 3: This is similar to case 2 except that the SSSC is used for control of the reactive power flow of line 12-15. The reactive power flow control reference is $Q_{15,12}^{Spec} = -1$ MVar.

Case 4: This is similar to case 2 except that the SSSC is used for control of the voltage magnitude at bus 15, and the voltage control reference is $V_{15}^{Spec} = 1.0$ p.u.

Case 5: This is similar to case 2 except that the SSSC is controlled to generate an equivalent reactance with a capacitive reactance control reference $X_{comp}^{Spec} = -0.2$ p.u.

Case 6: This is similar to case 1 except that there are three SSSCs installed on lines 12-15, 10-21 and 6-2, respectively. These SSSCs are used for control of the voltage at bus 15, reactive power flow of line 10-21, and active power flow of line 6-2, respectively. The control references are $V_{15}^{Spec} = 1.0$ p.u., $Q_{21,10}^{Spec} = -5$ MVar, and $P_{2,6}^{Spec} = 45$ MW, respectively.

Case 7: This is similar to case 6 except that the SSSC on line 6-2 is controlled to generate an equivalent reactance with respect to the capacitive reactance control reference $X_{comp}^{Spec} = -0.1$ p.u., and the SSSC on line 12-15 is used to control the active power flow of that line.

The results of cases 1–7 are summarized in Table 2.5.

Table 2.5. Results of the IEEE 30 bus system

Case No.	Solutions of the SSSCs parameters	Number of iterations
Case 1	None	4
Case 2	$\theta e_{12,15} = -109.64^\circ$, $Vse_{12,15} = 0.06060 p.u.$	5
Case 3	$\theta e_{12,15} = -97.03^\circ$, $Vse_{12,15} = 0.09639 p.u.$	5
Case 4	$\theta e_{12,15} = -101.78^\circ$, $Vse_{12,15} = 0.08344 p.u.$	6
Case 5	$\theta e_{12,15} = -109.93^\circ$, $Vse_{12,15} = 0.05972 p.u.$	5
Case 6	$\theta e_{12,15} = -101.05^\circ$, $Vse_{12,15} = 0.08577 p.u.$ $\theta e_{10,21} = -105.13^\circ$, $Vse_{10,21} = 0.05787 p.u.$ $\theta e_{6,2} = 75.77^\circ$, $Vse_{6,2} = 0.03539 p.u.$	6
Case 7	$\theta e_{12,15} = -108.02^\circ$, $Vse_{12,15} = 0.06663 p.u.$ $\theta e_{10,21} = -105.27^\circ$, $Vse_{10,21} = 0.05538 p.u.$ $\theta e_{6,2} = 77.79^\circ$, $Vse_{6,2} = 0.04878 p.u.$	5

In these cases above and the following discussions, the control references of active and reactive power flows are referred to P_{ji}^{Spec} , Q_{ji}^{Spec} , which are at the sending end of a transmission line. Active power flow and reactive power flows at the sending end of the line are referred to $-P_{ji}$, $-Q_{ji}$ since the sending end of the line is connected to the SSSC bus j .

The test results of the IEEE 118 bus system are described as follows,

Case 8: This is a base case of the IEEE 118 bus system.

Case 9: This is similar to case 8 except that there are three SSSCs installed for control of active power flow of line 21-20, reactive power flow of line 45-44 and voltage of bus 95, respectively.

Case 10: This is similar to case 9 except that the SSSC of line 94-95 is used to control the reactance of that line.

The test results of cases 8-10 are given by Table 2.6.

Table 2.6. Results of the IEEE 118 bus system

Case No.	Number of SSSCs	Number of iterations
Case 8	None	4
Case 9	3	5
Case 10	3	5

Further cases are carried out on the IEEE 300 bus system, which are as follows,

Case 11: This is a base case of the IEEE 300 bus system

Case 12: Similar to case 11 except that there are four SSSCs installed. The first SSSC is installed for control of the reactive power flow of line 198-197. The second SSSC is used for control of the voltage of the SSSC at bus 49. The third SSSC is installed for control of the reactance of line 126-132. The fourth SSSC is used for control of the active power flow of line 140-137.

Table 2.7. Results of the IEEE 300 bus system

Case No.	Number of SSSCs	Number of iterations
Case 11	None	6
Case 12	4	7

From the results of Table 2.5 to Table 2.7, it can be seen that, in comparison with those cases of base power flow solutions, the Newton power flow solutions with the SSSCs need more iterations but can converge within 7 iterations with the convergence tolerance of $1e-12$ p.u. ($1e-10$ MW/MVar). The convergence characteristics of case 11 without the SSSCs and case 12 with four SSSCs on the IEEE 300-bus system are shown in Fig. 2.5. The quadratic convergence characteristics of the Newton's algorithm can be clearly observed.

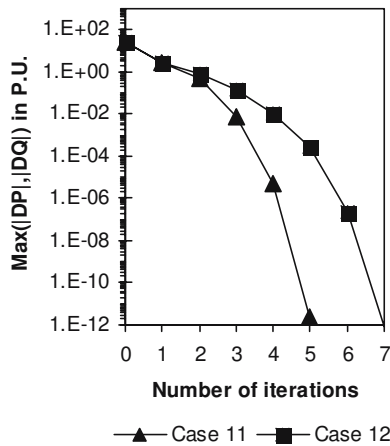


Fig. 2.5. Power mismatches as function of number of iterations

2.3.3.2 Enforcement of Voltage and Current Constraint of the SSSC

In the following, examples of the enforcement of the voltage and current constraints of the SSSC are given:

Case 13: This is similar to case 3 except that a voltage limit is applied to the SSSC.

Case 14: This is similar to case 2 except that a current limit is applied to the SSSC.

The test results of cases 13 and 14 are presented in Table 2.8. Test results of constraints enforcement of SSSC on the IEEE 118-bus system and the IEEE 300-bus system can be found in [34].

Table 2.8. Results of constraints enforcement of the SSSC for the IEEE 30 bus system

Case No.	Actual voltage or current of the SSSC without constraint enforcement	Voltage or current limits of the SSSC for constraints enforcement	Number of iterations
Case 13	$Vse_{12,15} = 0.09639 p.u.$	$Vse_{12,15}^{\max} = 0.08 p.u.$	6
Case 14	$Ise_{15,12} = 0.3 p.u.$	$Ise_{15,12}^{\max} = 0.25 p.u.$	7

This section has introduced a multi-control functional model for the Static Synchronous Series Compensator (SSSC) suitable for power flow analysis. The model has explored the multi-control options of the SSSC such as (a) the active power flow on the transmission line, (b) the reactive power flow on the transmission line, (c) the bus voltage, and (d) the impedance (precisely reactance) of the transmission line, etc. Furthermore, within the model, the operating voltage and current constraints of the SSSC have been fully considered. Detailed implementation of the novel multi-control functional model in the Newton power flow algorithm has been presented.

2.4 Modeling of SVC and TCSC in Power Flow Analysis

In the previous sections, the mathematical models of converter based FACTS devices have been discussed in details. The models proposed will be of great importance to develop production grade power flow programs. In this section, the modeling of SVC and TCSC will be addressed. Traditionally SVC and TCSC are considered as variable susceptance and reactance, respectively, in power flow analysis. In this section, SVC and TCSC will be equivalently represented as STATCOM and SSSC, respectively. Then SVC and TCSC can be incorporated in the Newton power flow program very easily.

2.4.1 Representation of SVC by STATCOM in Power Flow Analysis

SVC can provide voltage and reactive power control by varying its shunt reactance [33]. Two popular configurations of SVC are the combination of fixed capacitor and Thyristor Controlled Reactor (TCR) and the combination of Thyristor Switched Capacitor (TSC) and TCR. A SVC consisting of a fixed capacitor and a TCR is shown in Fig. 2.6a.

In power flow analysis, the total susceptance of the SVC may be taken as a variable and additional voltage or reactive power control equation should be included. In contrast to the SVC models used in power flow analysis, the SVC will be implemented in the way as for a STATCOM. So at first the SVC Fig. 2.6a should be converted into an equivalent STATCOM in the point of view of power flow analysis. The equivalent representation of the SVC is illustrated by Fig. 2.6b.

In Fig. 2.6b Z_{sh} is given by:

$$Z_{sh} = j(X_{TCR}^{\min} + X_{TCR}^{\max})/2 \quad (2.61)$$

where $X_{TCR}^{\min}$, $X_{TCR}^{\max}$ are the lower and upper limits of the variable reactance of the TCR branch in Fig. 2.6. Now the variable reactance of the TCR branch in Fig. 2.6a can be equivalently represented by an impedance Z_{sh} in series with a variable voltage source V_{sh} which can only inject reactive power into bus i . The equivalent of the branch is identical to that of STATCOM.

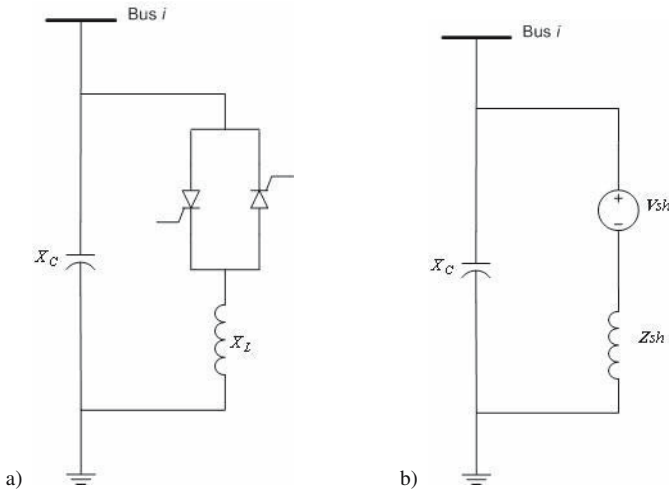


Fig. 2.6. a) A SVC with a fixed capacitor and TCR, b) Equivalent representation of SVC by STATCOM for power flow analysis

Hence, the STATCOM model can be applied directly to the SVC except that the following inequality should hold instead of (2.18):

$$X_{TCR}^{\min} \leq X_{TCR} \leq X_{TCR}^{\max} \quad (2.62)$$

where X_{TCR} is given by:

$$X_{TCR} = |V_{sh} Z_{sh} / (V_i - V_{sh})| \quad (2.63)$$

2.4.2 Representation of TCSC by SSSC in Power Flow Analysis

A typical TCSC, as shown in Fig. 2.7, can provide continuous control of power on the AC line with a variable series capacitive reactance. The TCSC consists of a fixed capacitor in parallel with a Thyristor Controlled Reactor (TCR). In principle, a TCSC is very similar to a SVC. The difference between them is that the former is usually series connected with a transmission line while the latter is usually shunt connected with a local bus.

Similarly, a TCSC can be represented by an equivalent SSSC for power flow analysis.

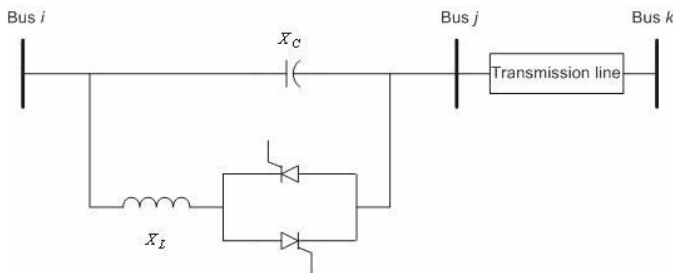


Fig. 2.7. A typical TCSC

In this section, the representation of SVC and TCSC as STATCOM and SSSC, respectively has been introduced. With this approach SVC and TCSC can be very easily incorporated into the Newton power flow program with just minor modifications of computer code.

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